Metrology with Atomic Clocks

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Part I: Introduction and review:

Atomic clocks and applications

Part II: A modern atomic clock:
The mercury-ion optical clock

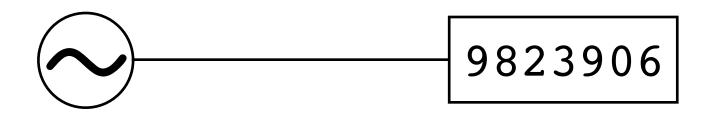




What is a clock?

An Oscillator (Generates periodic events)

A Counter (Counts and displays events)



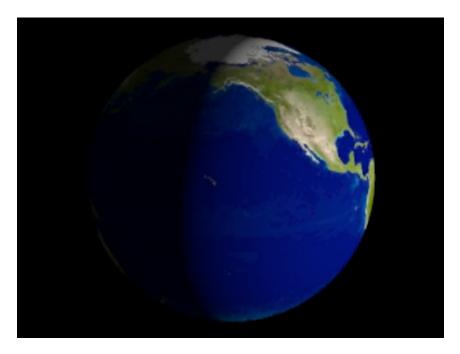
Period Δt Frequency $v=1/\Delta t$

Count time intervals



What is a clock?

An Oscillator (Generates periodic events)



 $\Delta t = 1 \text{ day}$ $v \sim 10^{-5} \text{ Hz}$

A Counter (Counts and displays events)

		Α	ugust 200	14		
Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	1	2	33	4



What is a clock?

An Oscillator

A Counter (Generates periodic events) (Counts and displays events)

Other canonical examples:

Mechanical pendulum ———— Gears and clock face

Quartz crystal Electronic counter

Now: What makes a clock a good time standard?



What makes a time standard?

- For a standard, want clock with high stability and accuracy
 - Stability: ability to produce identical successive time intervals

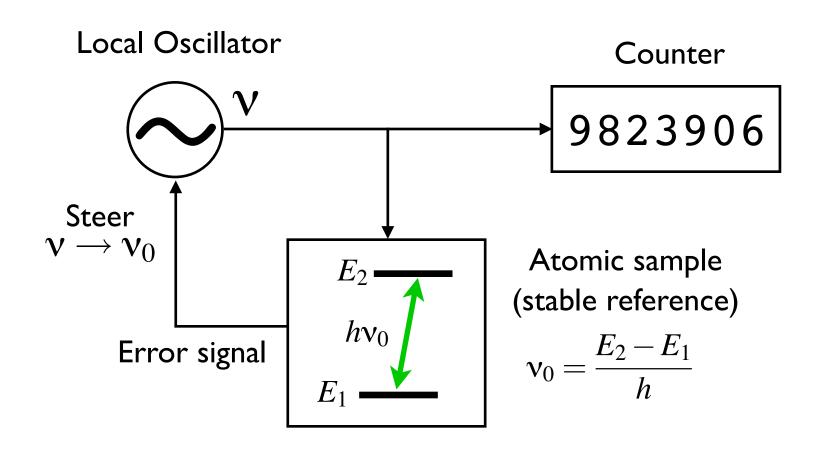
$$\Delta t_i = \Delta t_j, \ orall \ i.e., \qquad rac{d
u}{d t} \longrightarrow 0$$

• Accuracy: ability to produce the same frequency as other clocks

$${f v} \longrightarrow {f v}_0$$
 i.e., $\Delta t_i \longrightarrow {1\over {f v}_0}$

Accuracy requires an additional ingredient: stable (unperturbed) reference

Basic idea of an atomic clock





Characterizing clock performance

• Stability characterized by Allan deviation $\sigma_y(\tau)$ for averaging time τ , with frequency fluctuations $\Delta \nu_{rms}$.

$$\sigma_{\mathrm{y}}(au) pprox \left\langle rac{\Delta
u_{\mathrm{rms}}}{
u_{0}}
ight
angle_{ au}$$

Quantum projection noise limit for an atomic clock:

$$\sigma_{y}(au) pprox rac{1}{2\pi
u_{0} \sqrt{NT_{
m R} au}}$$

 $T_{\rm R}$: (Ramsey) Interrogation time

N: Number of atoms

 Limit to accuracy: uncertainty in correcting for changes in transition frequency relative to that of unperturbed atoms

Confinement

Families of atomic clocks

Transition frequency

Vapor Cell/ weak trap

Beam/fountain/cold atoms

Tight confinement

Microwave	Optical	
H Maser Compact Cs, Rb Hg+ (large N)		
Cesium, rubidium	Calcium	
Hg+ (small N)	Single ion clocks, Optical lattice clocks	



Microwave frequency clocks

- Includes all "traditional" and commercial atomic clocks
- Based on ground-state hyperfine transitions:

Species	v_0 (GHz)
Н	1.420 405 751 770(3)
Rb	6.834 682 610 904 29(9)
Cs	9.192 631 770 (exact)
Hg ⁺	40.507 347 996 841 6(4)

• Several different technologies to probe these transitions

Microwave frequency clocks

Vapor Cell/ weak trap H Maser
Hg+ (large N)
Compact Cs, Rb

Trade accuracy for high stability or compactness

Fountains

Cesium, rubidium

Can achieve good stability and accuracy

Tight confinement

Hg+ (small N)

Trade stability for accuracy (Obsolete)



Cesium clocks

•Ground-state hyperfine transition defines SI second, all "absolute" frequency measurements are made vs. cesium.

•Fountain geometry maximizes probe time (~I s) stability can approach $\sigma_{v}(\tau)\sim 2\times 10^{-14}\tau^{-\frac{1}{2}}$

•Important systematics:

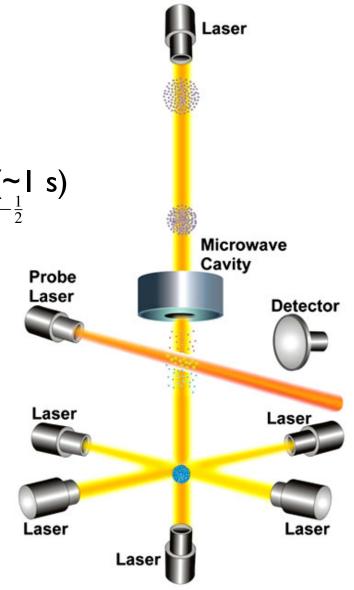
Doppler, 2nd order Doppler

Zeeman shifts

Blackbody radiation

Phase shifts in microwaves

Collisional shifts (lower for Rb)

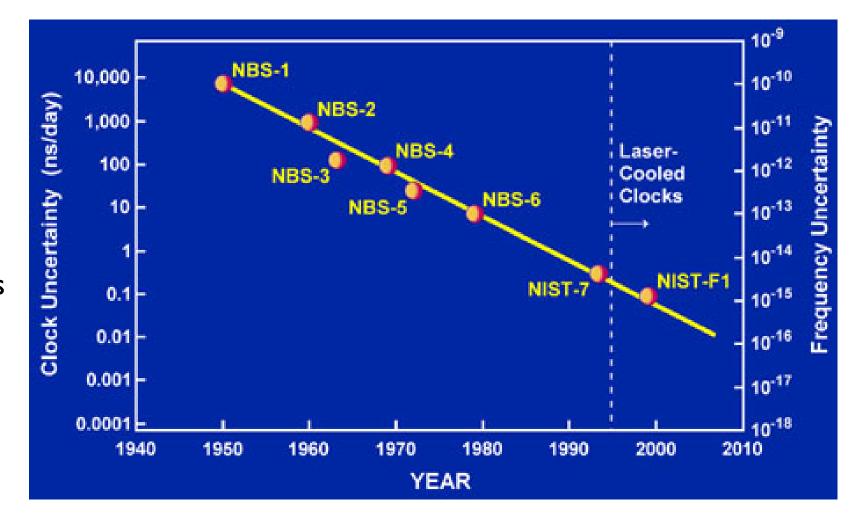




Cesium clocks

•Cesium is still the most accurate clock Fractional frequency uncertainty $\sim 1 \times 10^{-15}$

Is in 30 million years



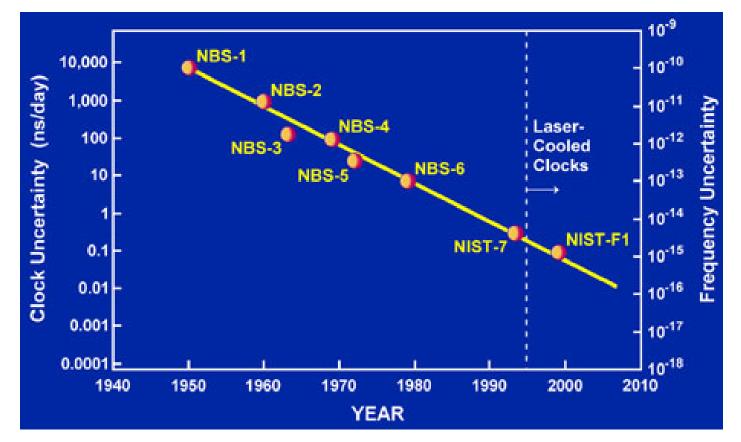
NIST Cs clocks

Cesium clocks

•Cesium is still the most accurate clock Fractional frequency uncertainty $\sim 1 \times 10^{-15}$

I s in 30 million years... or I ns in only II days.







Optical frequency clocks

- Higher transition frequency ⇒ higher stability
- Use optical transition instead of microwave

Recall stability limit:
$$\sigma_y(au) pprox \frac{1}{2\pi v_0 \sqrt{NT_{
m R} au}}$$

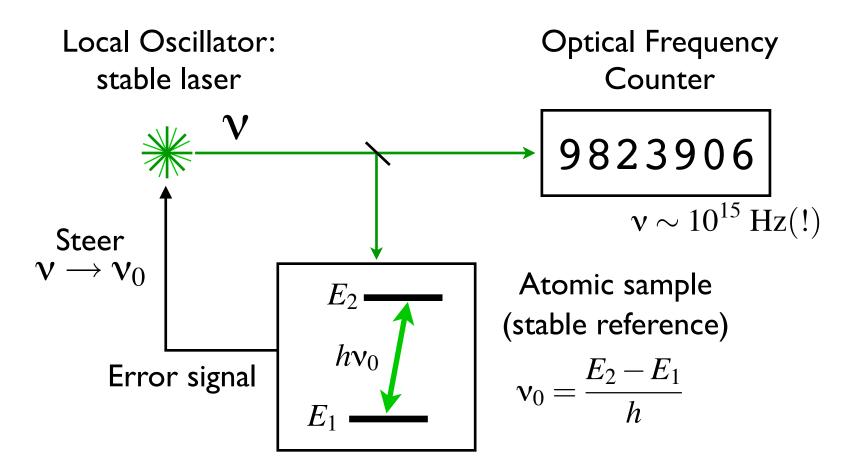
examples:

Cs fountain:
$$N=10^6$$
 atoms, $v_0=9$ GHz, $T_R=1$ s $\Longrightarrow \sigma_y(\tau)\sim 2\times 10^{-14}\tau^{-\frac{1}{2}}$

Hg+ optical clock:
$$N=1$$
 ion, $v_0=10^{15}$ Hz, $T_R=30$ ms $\Rightarrow \sigma_y(\tau) \sim 1 \times 10^{-15} \tau^{-\frac{1}{2}}$

- Increase in stability enables high accuracy for single ion.
- Extremely high stability for optical clock with large N.

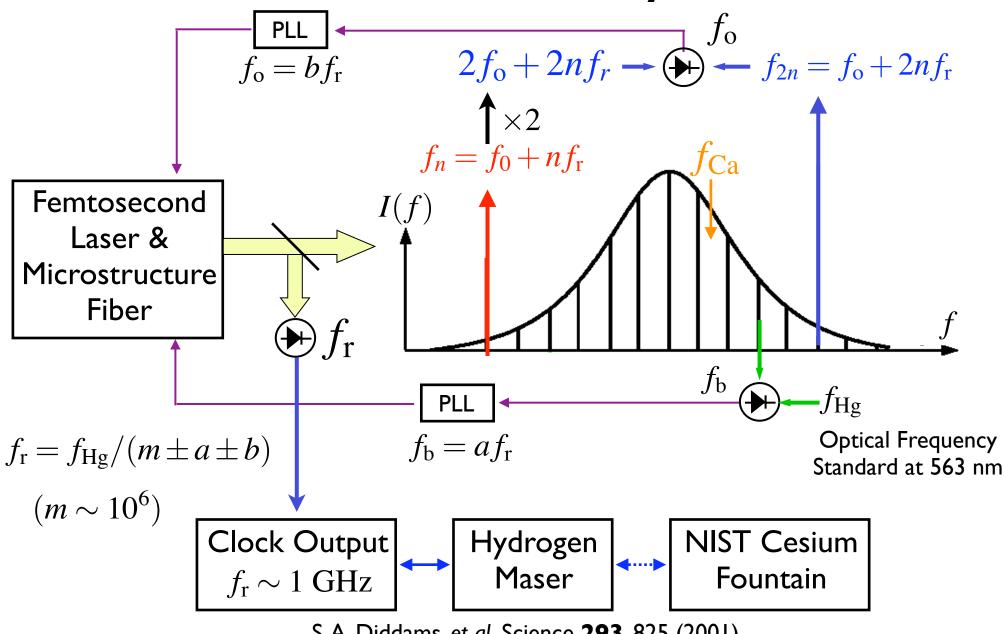
Optical frequency clocks



- Local oscillator is stable laser (not quartz crystal)
- Need to count frequency of light
 Only recently practical

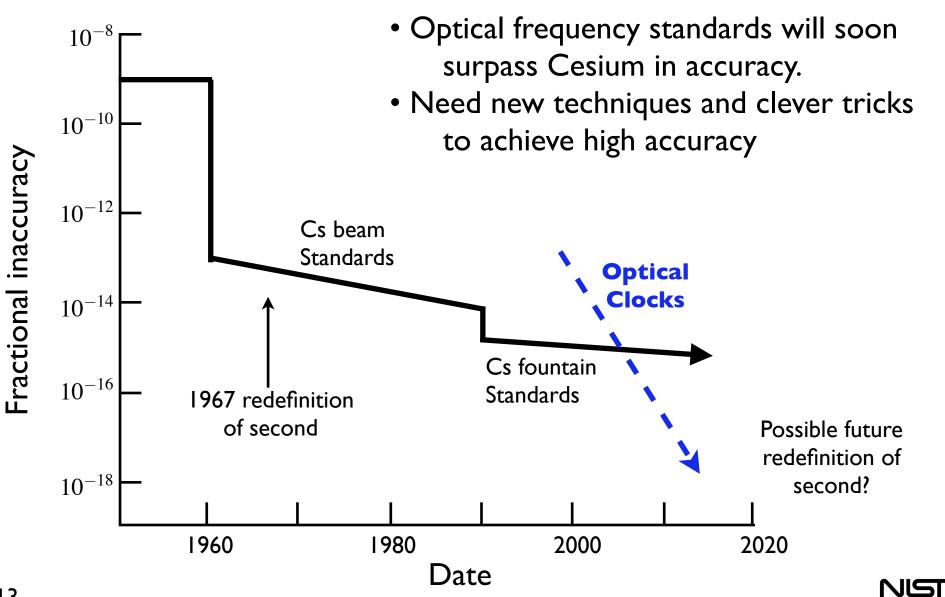


Femtosecond comb system



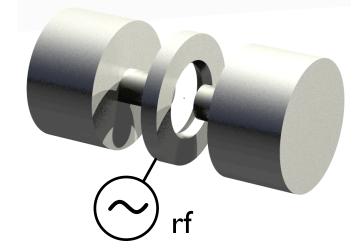
S.A. Diddams, et al. Science **293**, 825 (2001)

Optical frequency clocks



Trapped ions in an rf trap

- No static **E** or **B** fields.
- Trap acts on total charge of ion, not internal structure



- •Trap ion at pseudopotential minimum (trap center)
- ⇒ Trapping fields at ion location approach zero
- Can operate in tight-confinement (Lamb-Dicke) regime
- \Rightarrow First-order doppler free. 2nd-order doppler shift expected to limit accuracy $~\frac{\Delta\nu}{\nu_0}\sim 10^{-18}$
- Long storage times possible, ⇒ long probe times



Trapping neutral atoms

- An optical clock with large N might be expected to demonstrate high stability, approaching $\sigma_y(1~{\rm s})\sim 10^{-18}$ but high accuracy is harder.
- Traps for neutral atoms work by perturbing internal structure
 - Magnetic traps bad for clocks
 - -AC stark shifts in optical traps
- ⇒ Usually can only probe atoms in free-fall for a clock (Fountain, released from MOT, etc.)

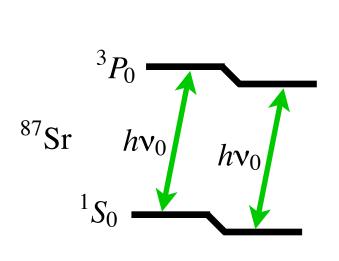
But, atoms move during free-fall

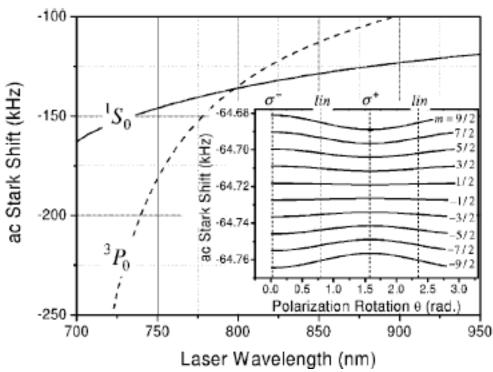
- Maximum probe time is limited
- Motion through probe beam can cause frequency shifts



The "Magic" wavelength

Clever idea (Katori*):Trap atoms in optical lattice at the "magic"
wavelength with equal light shift for the upper and lower states
of the clock transition: a "stark-free" optical lattice.





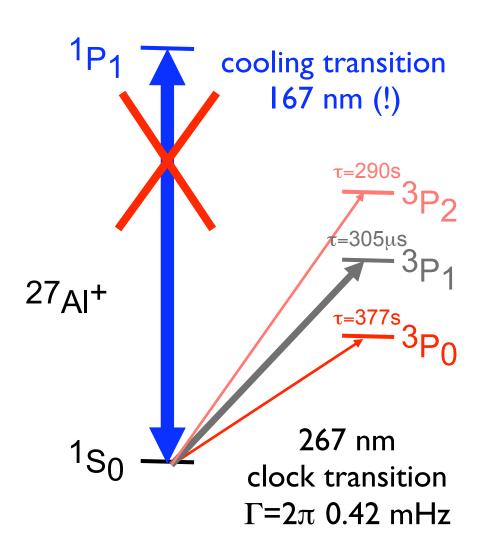
 Spectroscopy in tight-confinement (Lamb-Dicke) regime for neutral atoms; Expected to allow for high accuracy

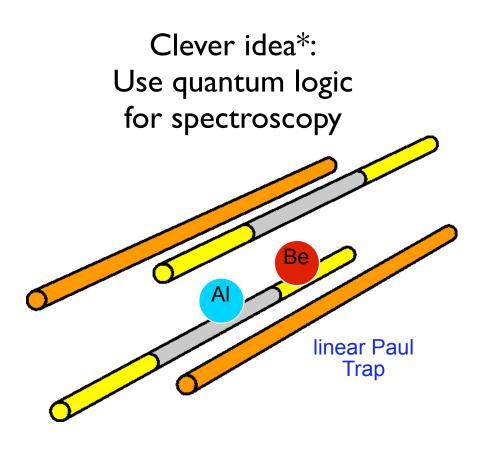


^{*} H. Katori, in *Proc. 6th Symposium Frequency Standards and Metrology*, Ed. P. Gill (World Scientific, Singapore, 2002), pp. 323-330.

The Al⁺clock at NIST

Piet Schmidt, Till Rosenband, W. M. Itano, D.J. Wineland, J.C. Bergquist





^{*} D.J. Wineland et al, in *Proc. 6th Symposium Frequency Standards and Metrology*, Ed. P. Gill (World Scientific, Singapore, 2002), pp. 361-367.



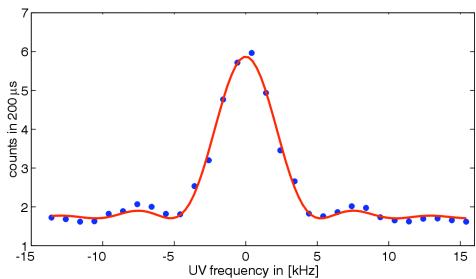
The Al⁺clock at NIST

Clock interrogation cycle:

- Sideband cooling of Be⁺ to ground state
- Interrogate AI⁺ clock transition
- Map internal state of Al⁺ ion onto motional mode
- Map motional mode onto Be⁺ internal state
- Read out Be⁺ internal state

Technique has been recently demonstrated on both wide and narrow Al transitions

$$^{1}S_{0} \leftrightarrow^{3} P_{1} F = 7/2$$





Optical standard candidates*

Ion	λ	Δv	Groups*
Sr ⁺	674 nm	0.4 Hz	NPL, NRC
In ⁺	236 nm	0.82 Hz	MPQ
Hg ⁺	282 nm	6.7 Hz	NIST
Al^+	267 nm	1 mHz	NIST
Yb ⁺	436 nm (S-D)	3.1 Hz	PTB
Yb ⁺	467 nm (<i>S-F</i>)	1 nHz	NPL

Atom	λ	$\Delta m v$	Groups*
Sr	698 nm	10 mHz	Tokyo, JILA, SYRTE
Ca	657 nm	470 Hz	NIST, PTB
Yb	578 nm	50 mHz	NIST, UW



Optical clock expectations

Ultimate limits:

Cold atoms

Calcium

High stability expected

Single ion clocks

High accuracy, moderate stability expected

Tight confinement

Optical lattice clocks

High accuracy and stability expected



Applications of atomic clocks

- Precision Measurement
 - -The second can be implemented more accurately than any other SI base unit
 - Commercial clock can measure frequency to 10⁻¹² in 1 s
 - Always try to cast measurements into frequency
 - Meter defined in terms of second
 - "Practical" standards for volt (Josephson voltage h/2e), and ohm (quantum hall resistance h/e²) depend on second
 - Implies "practical" ampere, kilogram based upon second
 - Other base units (candela, mole, kelvin) independent, cannot be measured precisely



Applications of atomic clocks

- Navigation and positioning:
 Satellite navigation and ranging; GPS, etc.
- Communication
 - Network synchronization
 - High-speed communication
- Astronomy
 - Very long baseline interferometry (VLBI)
- Fundamental tests
 - High-resolution spectroscopy
 - Astronomical relativity tests possible through VLBI
 - -Tests of quantum mechanics
 - Equivalence principle, relativity
 - Stability of fundamental constants



Partial error budget for NIST-F1 cesium fountain

Physical effect	Bias magnitude /10 ⁻¹⁵	Type B uncertainty /10 ⁻¹⁵
Second-order (quadratic) Zeeman	+44.76	0.3
Second-order Doppler	< 0.1	< 0.1
Cavity pulling	< 0.1	< 0.1
Rabi pulling	< 0.1	< 0.1
Cavity phase (distributed)	< 0.1	< 0.1
Fluorescent light shift	< 0.1	< 0.1
Adjacent atomic transitions	< 0.1	< 0.1
Spin exchange	0.0*(0.7 to 7.8)	0.84
Black body	-20.6	0.3
Gravitation	+180.54	0.1

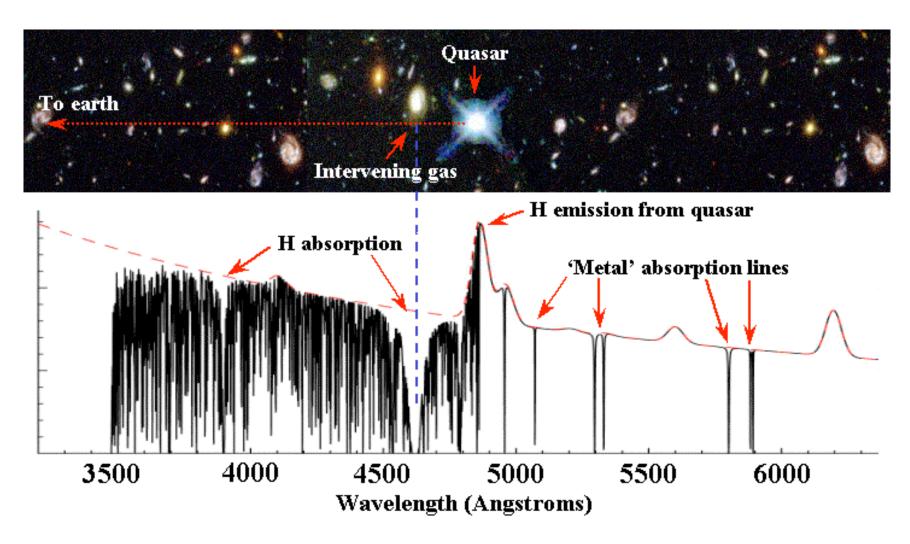
Stability of fundamental constants

Atomic clocks measure transition frequencies that generally depend upon a combination of fundamental constants.

- Some "historical" records to document values of fundamental constants in earlier universe:
 - Quasar absorption spectra at high redshift
 - Isotope ratios at Oklo fossil reactors
- Interpretation of data is model dependent; May indicate value of the fine structure constant $\alpha=e^2/\hbar c$ in earlier universe



Quasar absorption spectra



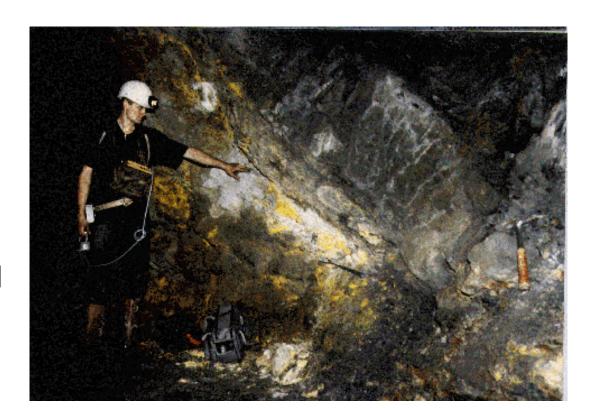
• Compare "metal" absorption lines to those measured in laboratory. (Precise laboratory values need work!)

Fossil reactors at Oklo

"The impression has been given that to design and construct a nuclear reactor is a feat unique to physical science and engineering creativity. It is chastening to find that, in the Proterozoic, an unassertive community of modest bacteria built a set of nuclear reactors that ran for millions of years."

- From James Lovelock, The Ages of Gaia (1988)

Remains of Oklo reactor core # 15, 1.5 billion years old





Stability of fundamental constants

	Yes, $lpha$ has changed.	No, $lpha$ has not changed
Oklo data 10 ⁹ yr	$\Delta\alpha(\text{past})/\alpha = +(4.5 \pm 1.5) \times 10^{-8}$	$\Delta\alpha(\text{past})/\alpha = -(0.36 \pm 1.44) \times 10^{-8}$
	S. K. Lamoreaux and J. R. Torgerson, Phys. Rev. D 69 121701 (2004).	Y. Fujii, et al., Nuc. Phys. B 573 , 377 (2000).
Quasar data	$\Delta\alpha(\text{past})/\alpha = -(0.54 \pm 0.12) \times 10^{-5}$	$\Delta\alpha(\text{past})/\alpha = -(6\pm6)\times10^{-7}$
10 ¹⁰ yr	M.T. Murphy, et al., astro-ph/0306483.	R. Srianand, et al, Phys. Rev. Lett. 92 121302 (2004).

•Clock comparisons may provide the best test of present-day changes in physical constants.



Conclusions for Part I

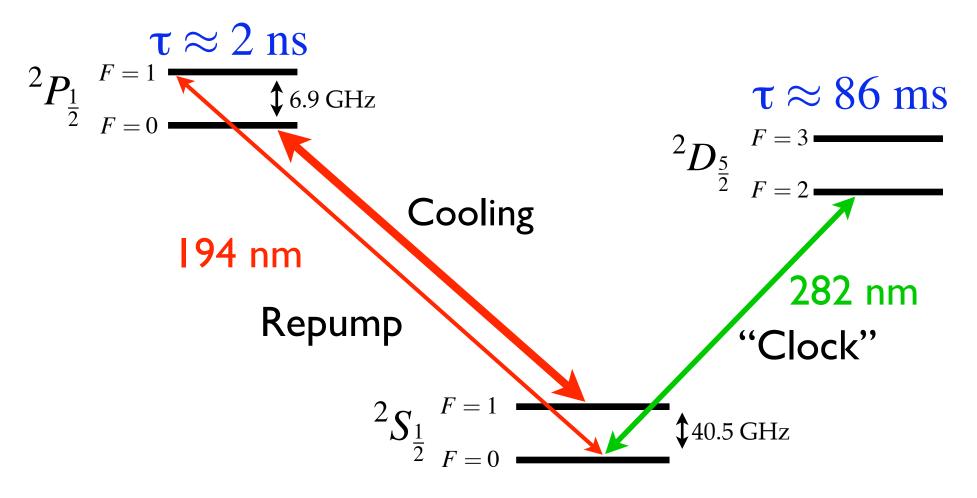
- The transition from microwave to optical transitions is facilitating a revolution in frequency standards
- New techniques in several areas are expected to lead to high accuracy, with wide benefits

Part II: the mercury ion clock

 An atomic clock based upon an optical transition in a single ion of mercury



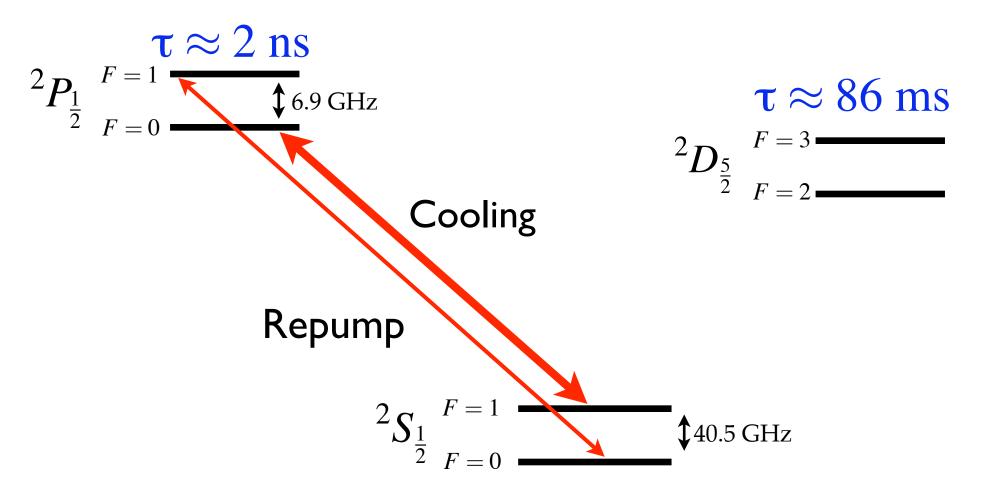
199 Hg⁺ Energy Levels



• Atomic line $Q \approx 1.5 \times 10^{14}$



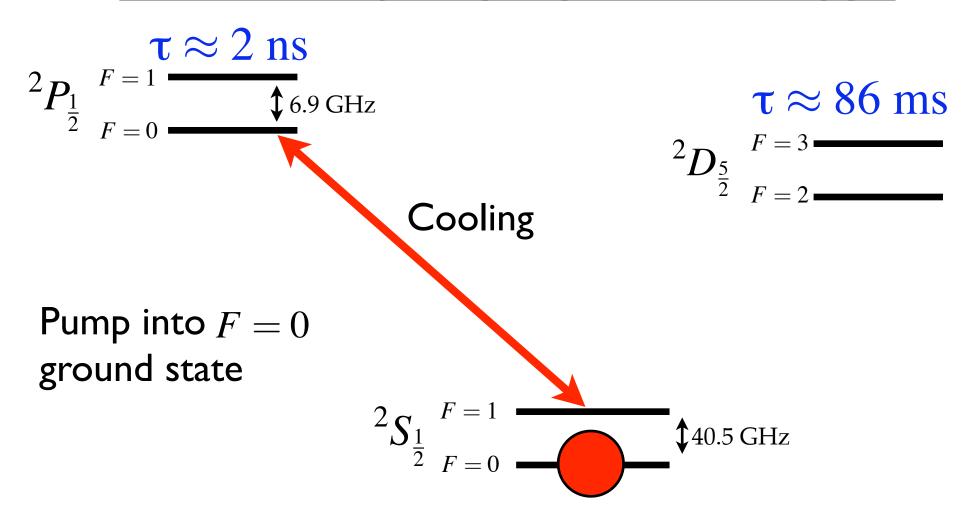
Quantum Jump Spectroscopy



•State detection by electron shelving H.G. Dehmelt, *Bull. Am. Phys. Soc.* **20**, 60 (1975).



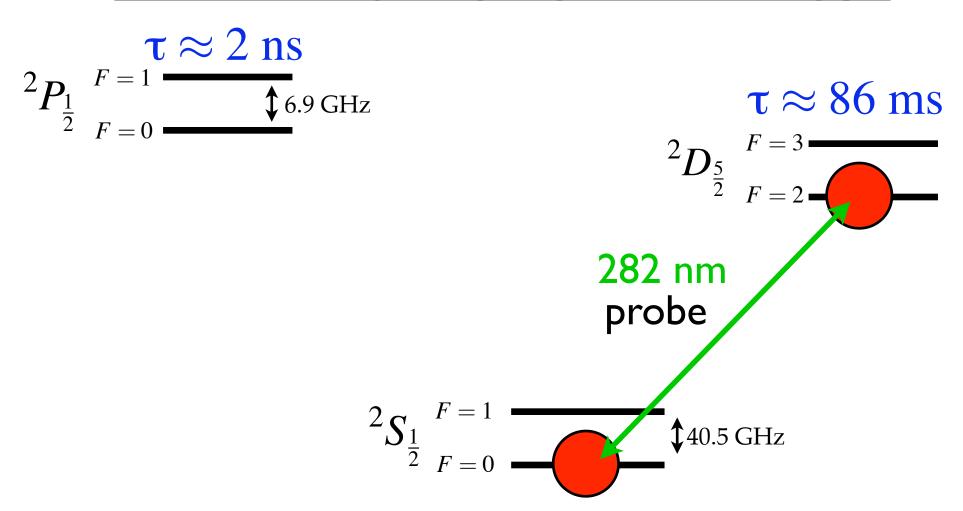
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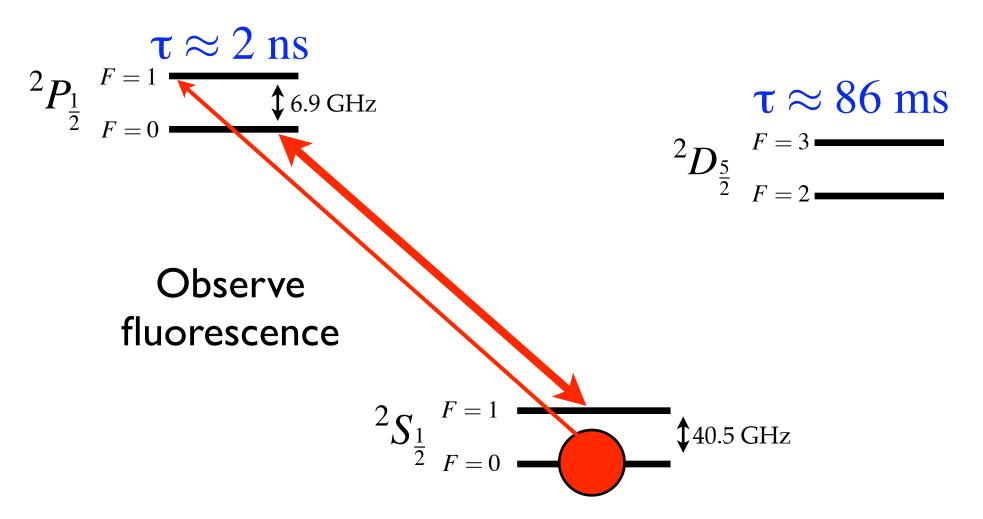
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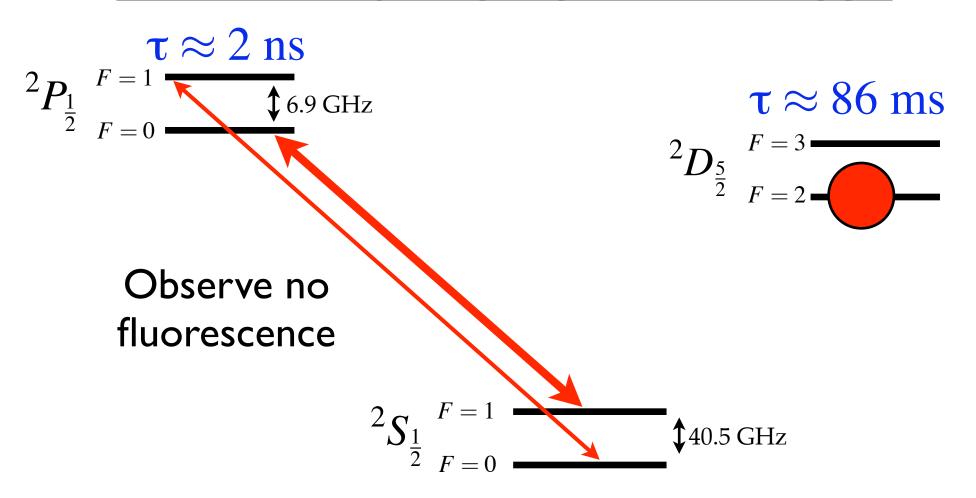
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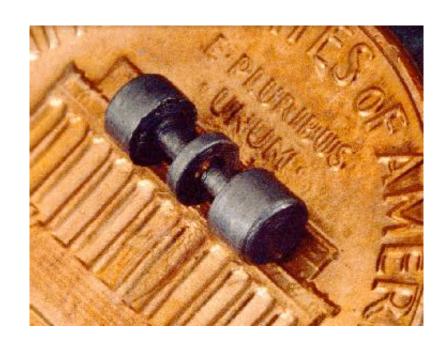
Quantum Jump Spectroscopy



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Cryogenic Spherical Paul Trap



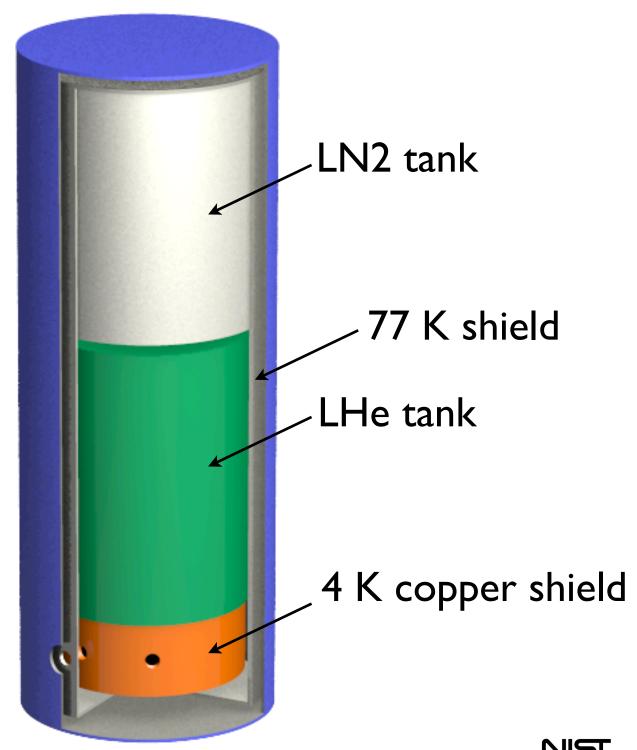
- Trap storage time: months (in principle)
- High environmental isolation (collision rate, blackbody)

Cryogenic trap system

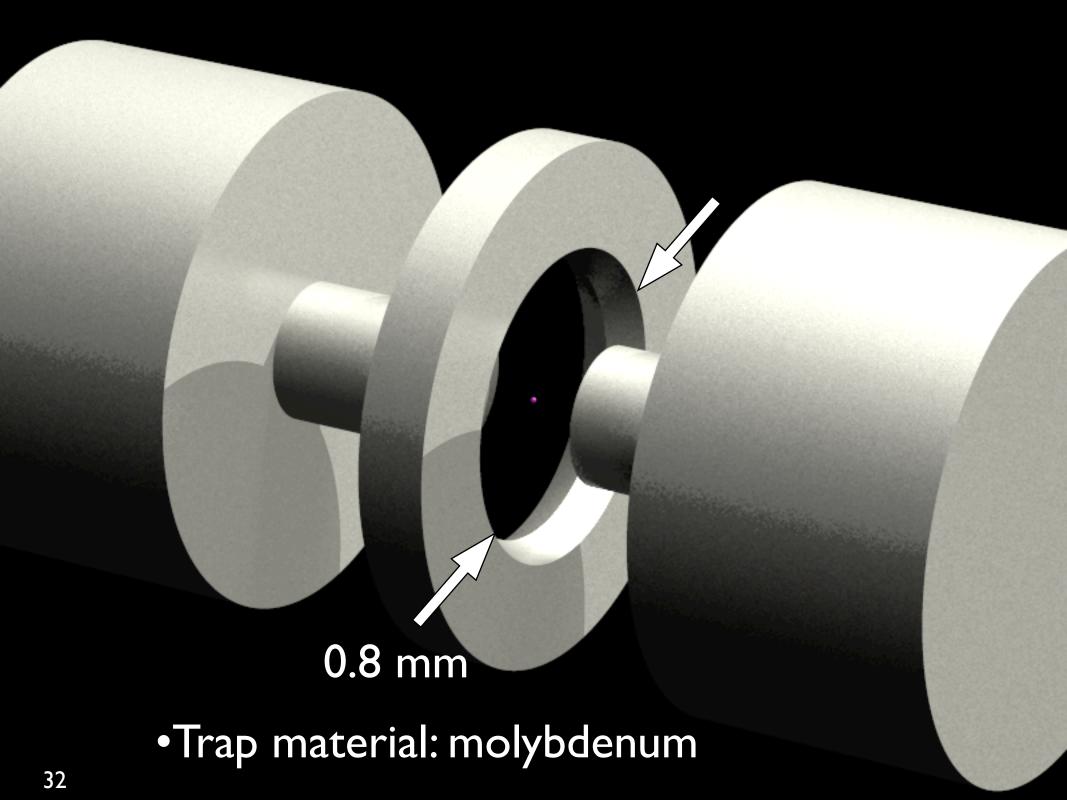


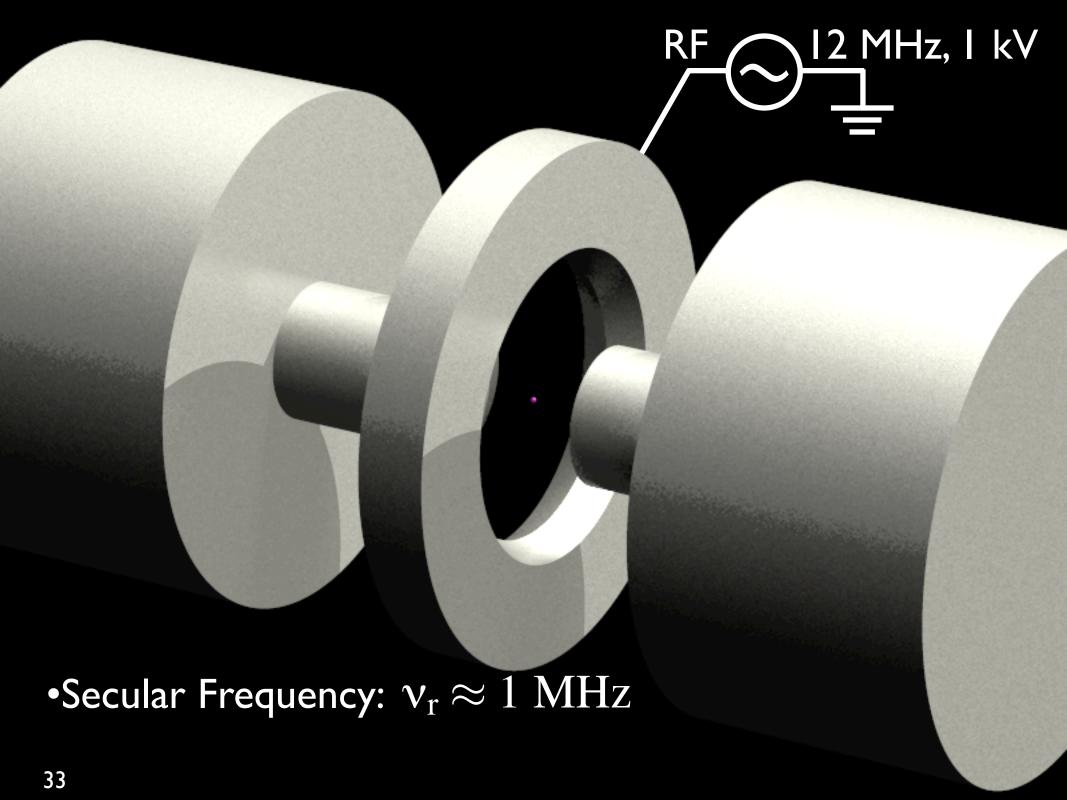


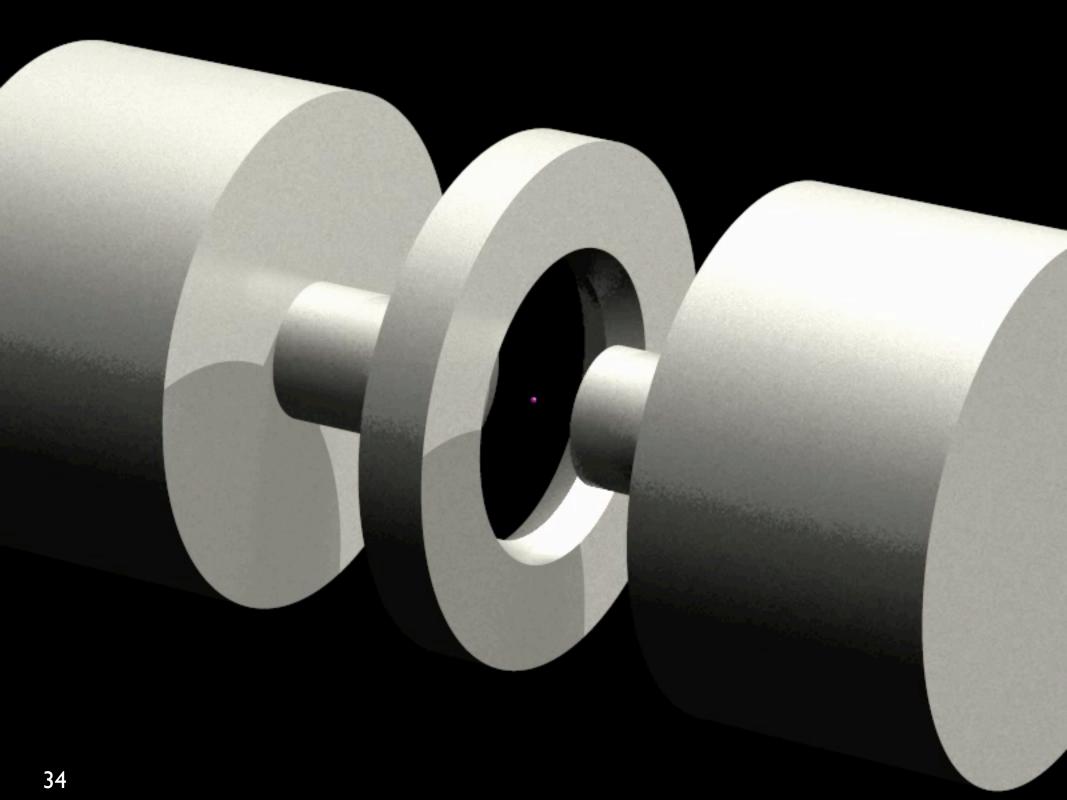
Cryogenic trap system



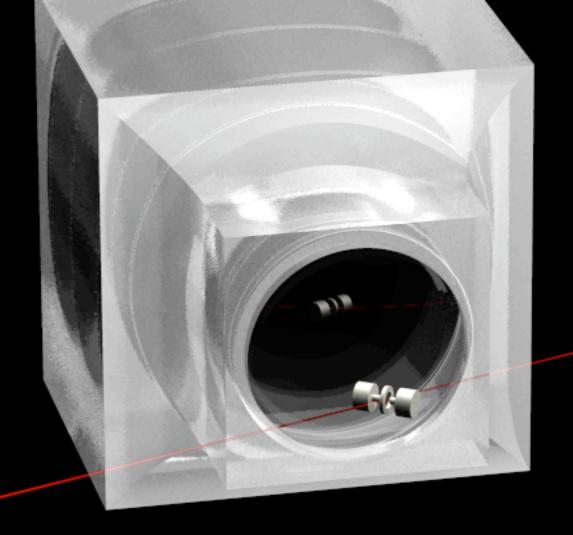












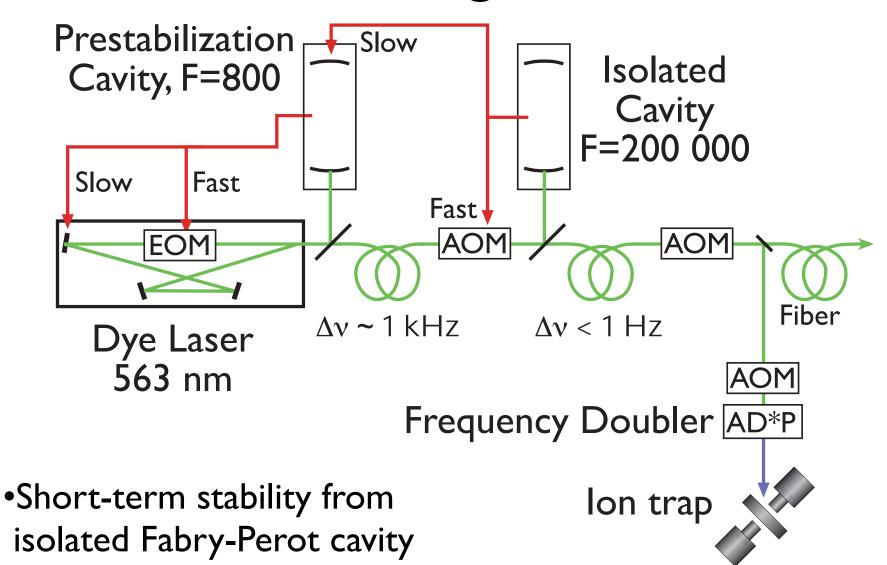
Fused silica lens in cryostat for state detection





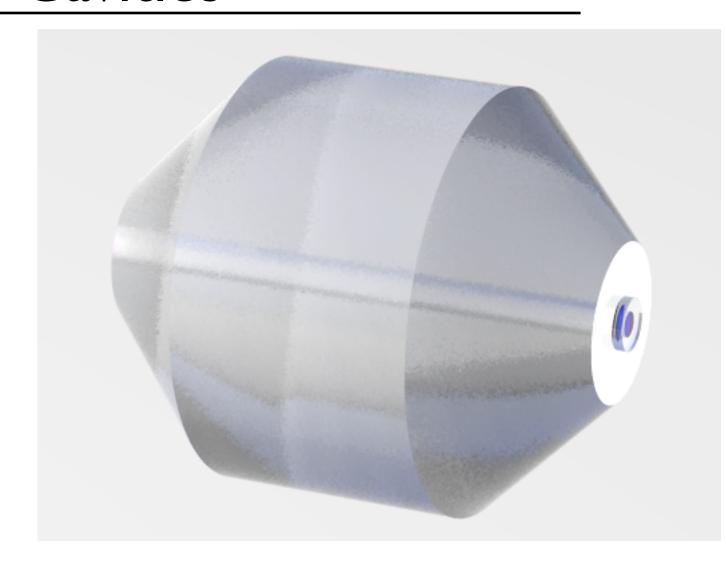
Probe Beam 282 nm

Stable 282 nm light source



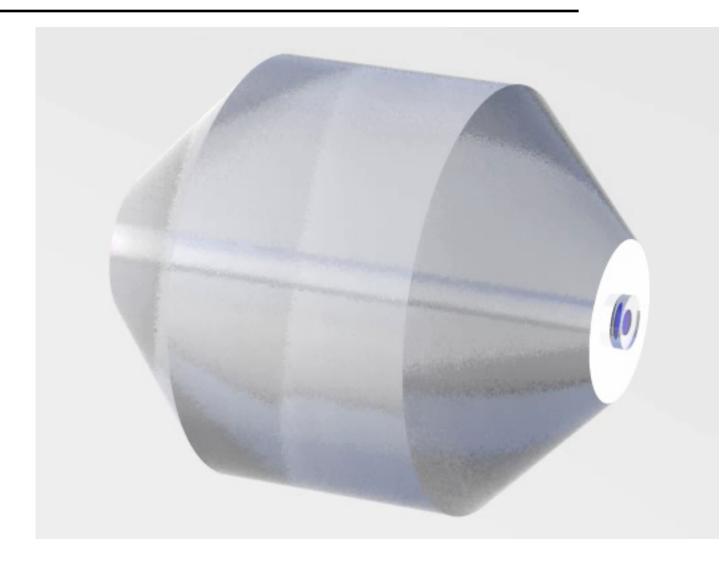
Long-term stability from ion

- ULE Spacer25 cm
- • $F \approx 200000$





- ULE Spacer25 cm
- • $F \approx 200000$





- ULE Spacer25 cm
- • $F \approx 200000$



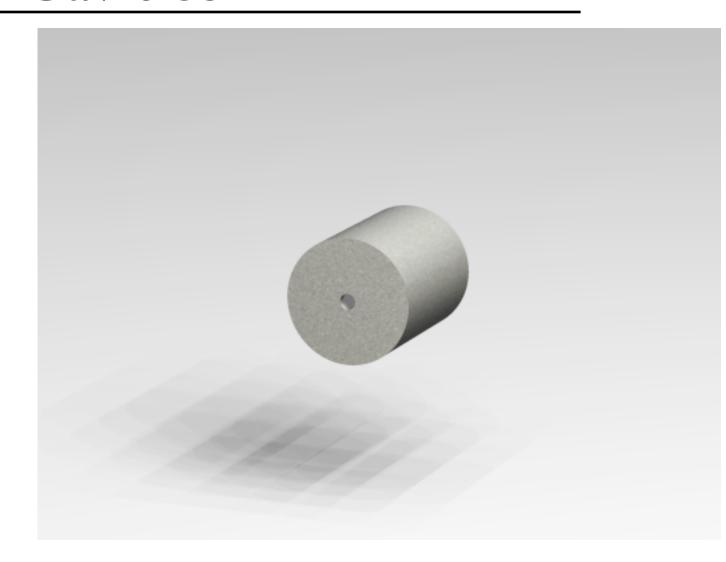


- ULE Spacer25 cm
- • $F \approx 200000$



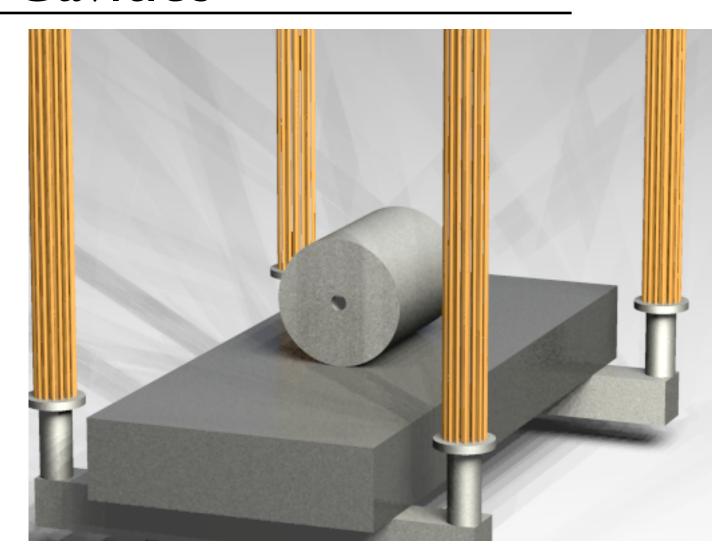


- ULE Spacer25 cm
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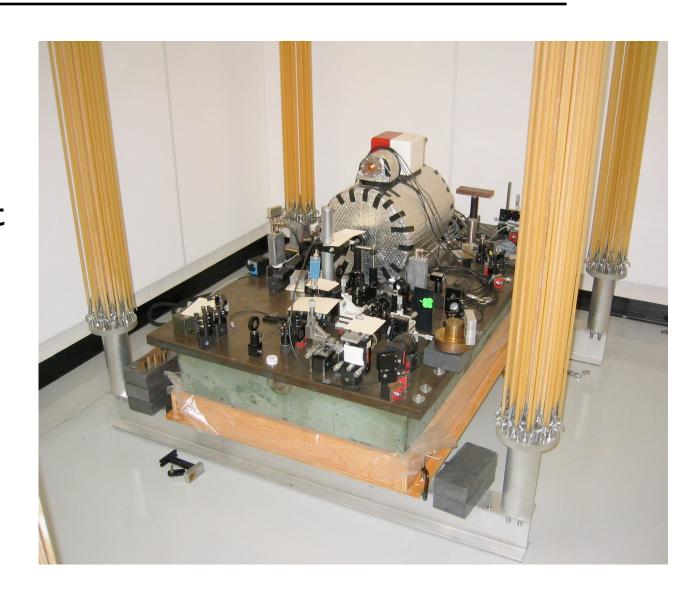


- ULE Spacer25 cm
- • $F \approx 200000$
- Thermal and seismic isolation



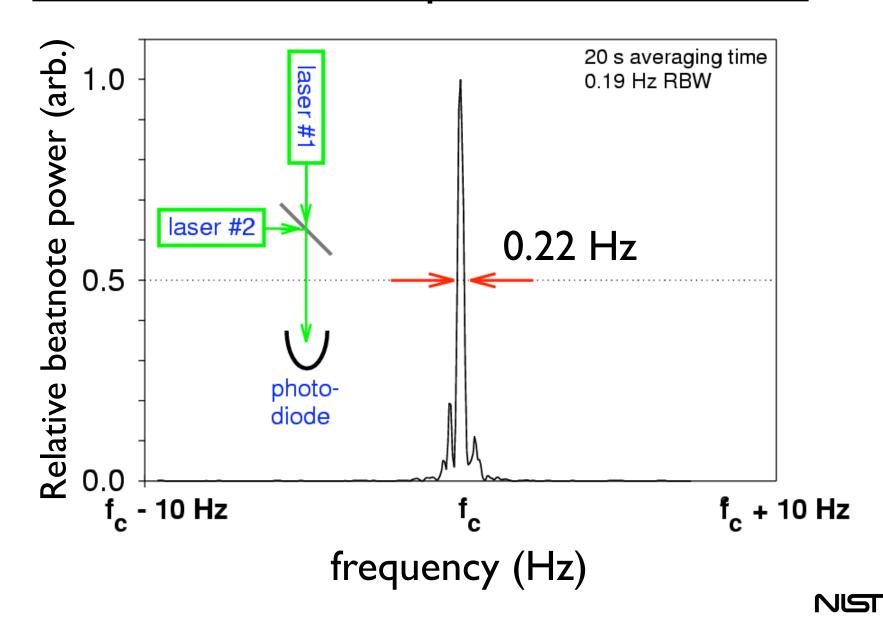


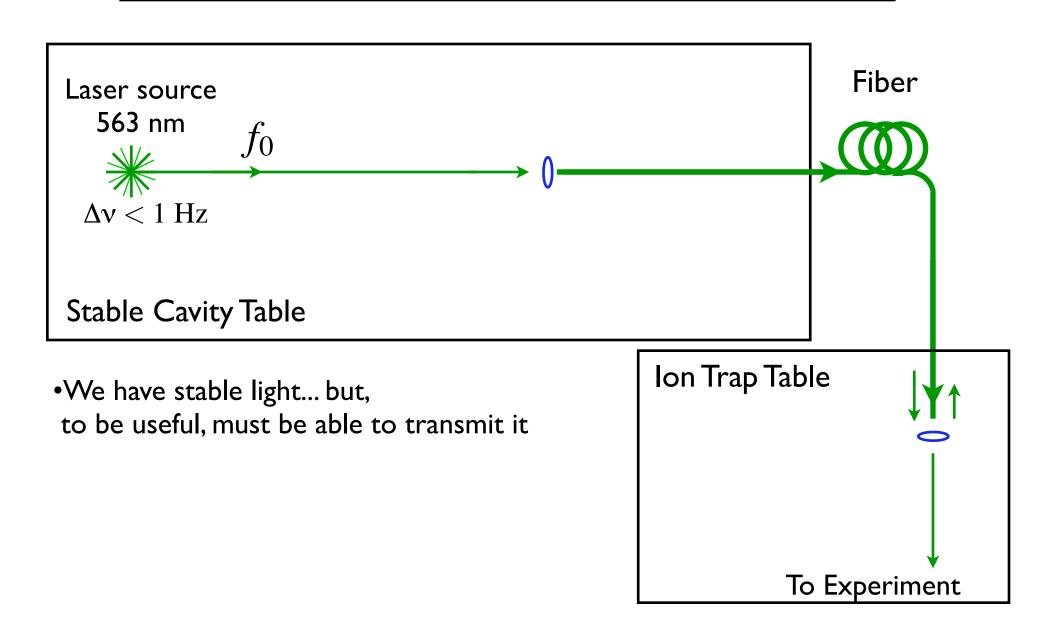
- Resonances near 0.3 Hz
- Servo table height by heating rubber tubing
- Two independent cavity systems

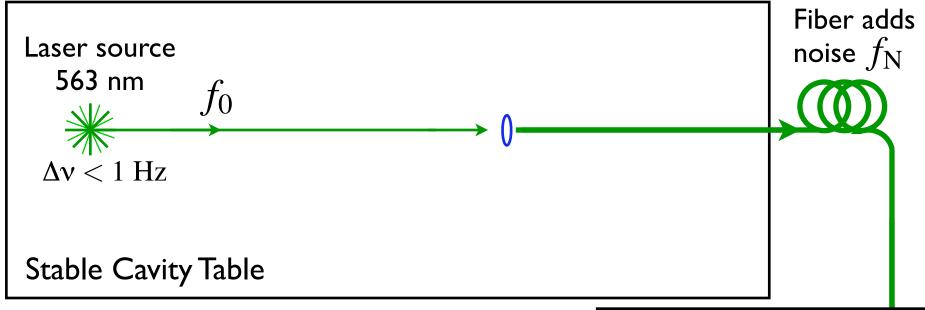




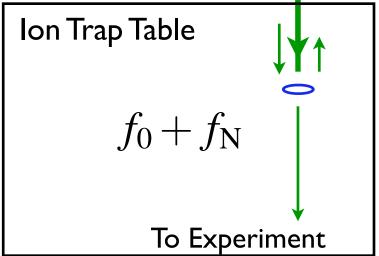
Beatnote between laser sources stabilized to independent cavities





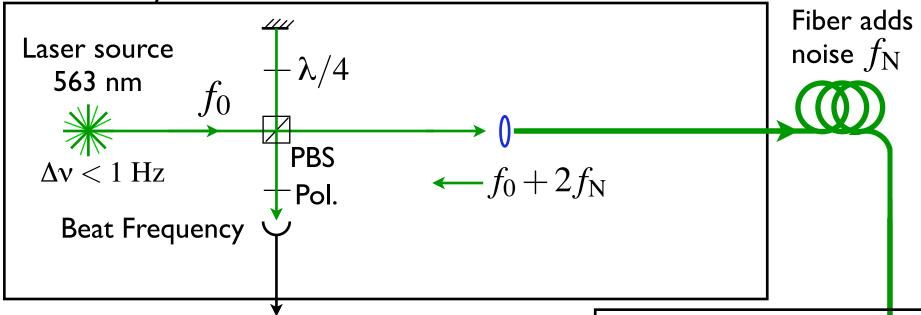


- We have stable light... but,
 to be useful, must be able to transmit it
- •Fiber noise broadens line by several kHz

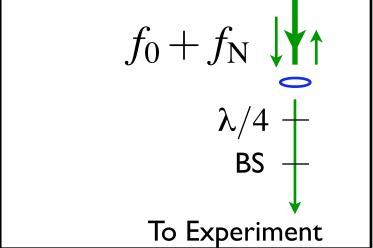




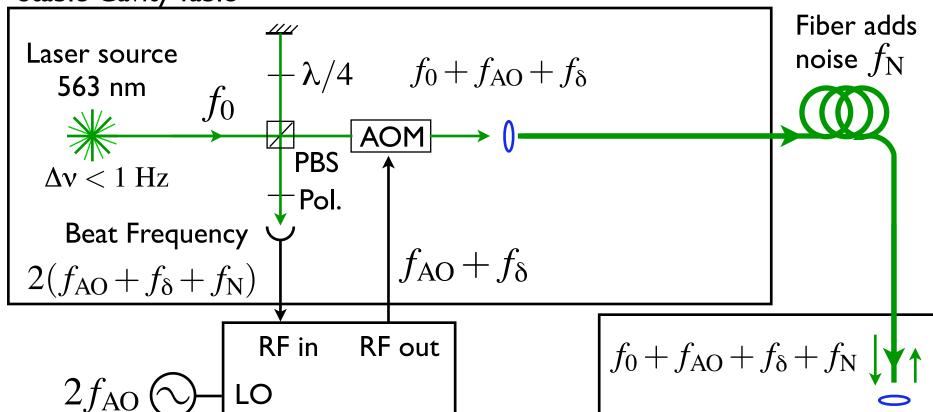
Stable Cavity Table



•Build interferometer with one arm along fiber path



Stable Cavity Table

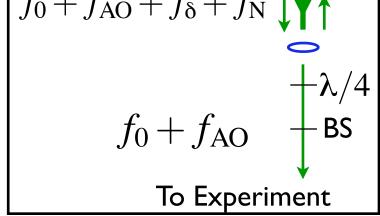


Lock $f_{\delta} = -f_{
m N}$

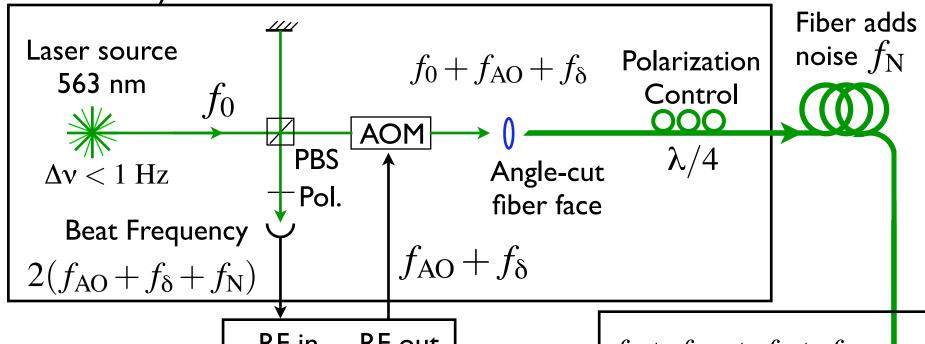
J. C. Bergquist, et al., Frontiers of Laser Spectroscopy, North Holland, 1994, p. 359. L.-S. Ma, P. Jungner, J. Ye, and J. L. Hall, Opt. Lett. 19, 1777 (1994).

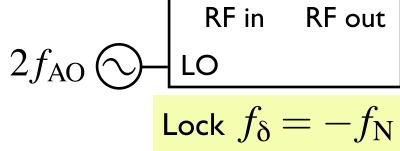
B. C. Young, et al., Proc. of the 14th Int. Conf. on Laser Spectroscopy,

World Scientific, 1999, p.61.



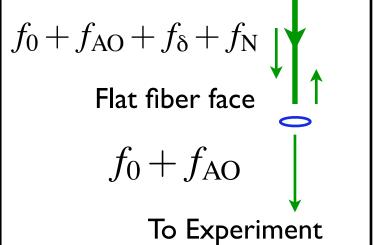






Simplification:

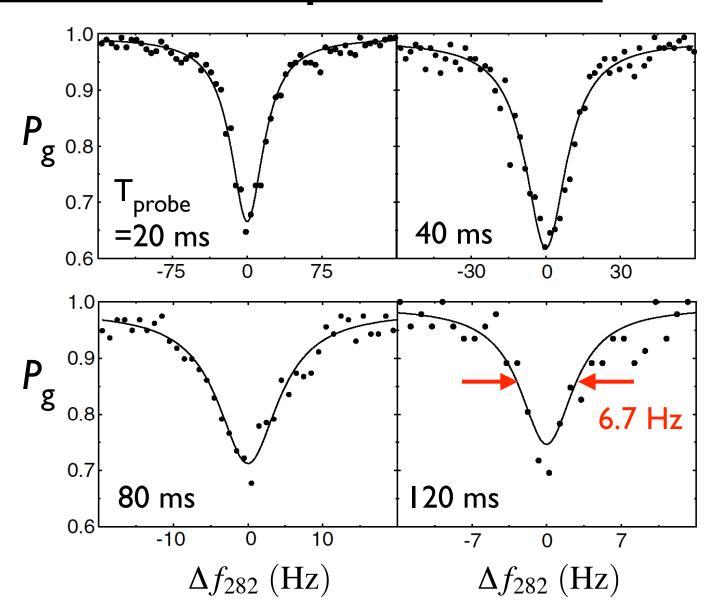
Remove waveplates & beamsplitter



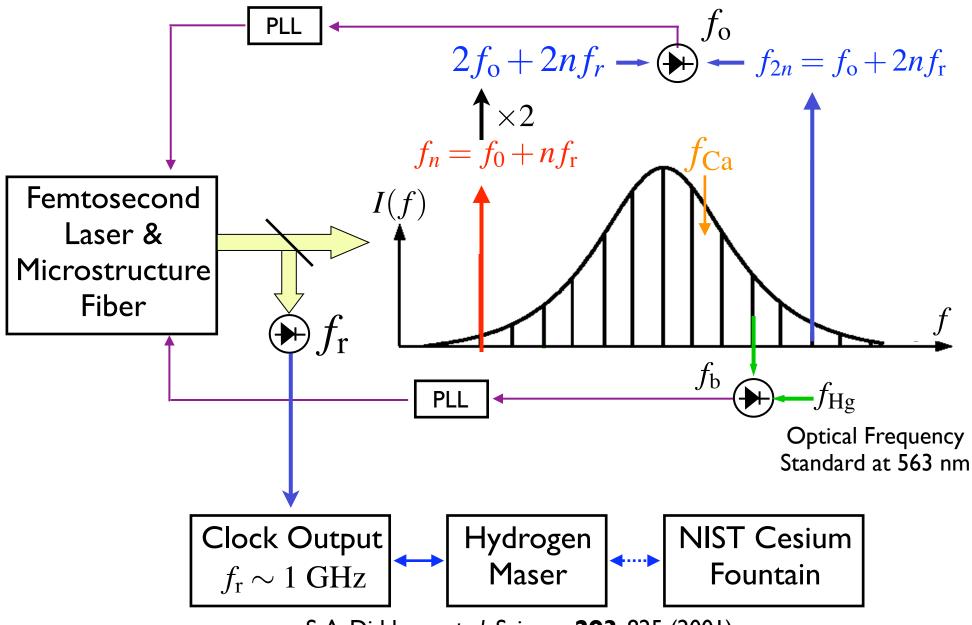
Absorption lineshapes

- Quantum-jump spectroscopy
- -Rabi Excitation
- -Transform-limited linewidth

•Lock to transition by stepping across transition.



Femtosecond comb system



S.A. Diddams, et al. Science 293, 825 (2001)

Systematic Frequency Shifts

Effect	Correction (Hz at 1.06 PHz)	Uncertainty (Hz)
2 nd order Zeeman (B-field uncertainty)	1700, (typ., daily)	2.8
2 nd order Zeeman (coefficient uncertainty)	0	2.6
$^2D_{rac{5}{2}}$ quadrupole shift	Not evaluated	10
H-maser frequency		4

Total fractional uncertainty: 10⁻¹⁴

•Negligible (at this level): ac stark shifts (laser, trap),

2nd order Doppler (micromotion, thermal),

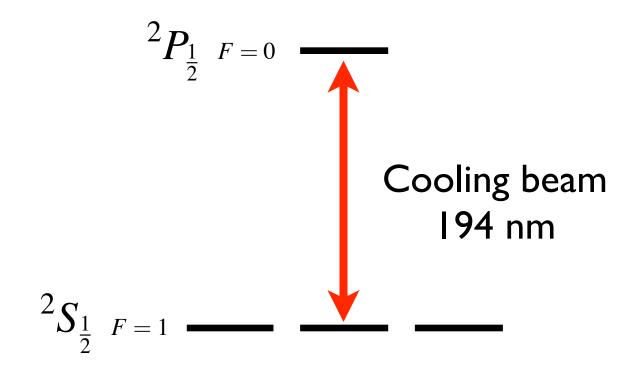
background collision shifts (helium),

blackbody shift, fs comb noise.

•All effects except quadrupole should reduce below 10-18



Avoiding dark states



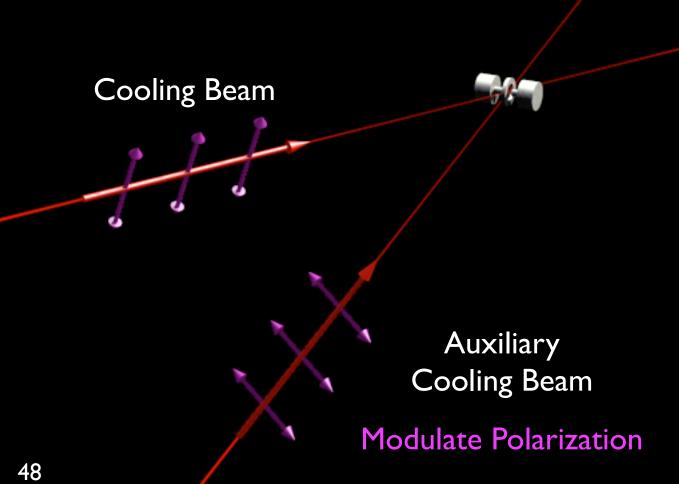
- Dark states for any polarization
- Limit fluorescence rate
- Destabilize with B-field or polarization modulation





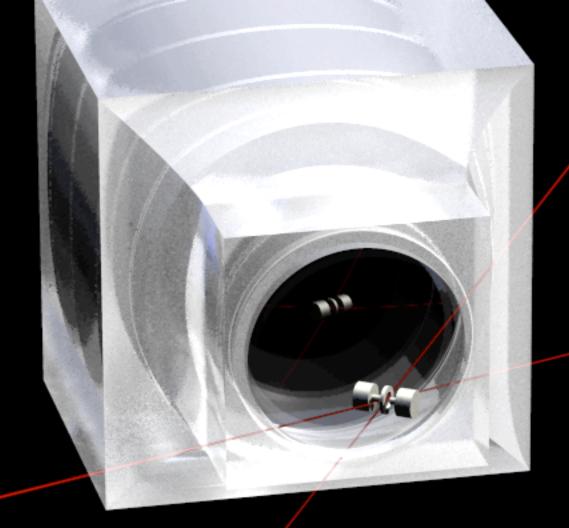


Auxiliary Cooling Beam



Auxiliary Cooling Beam

Modulate Polarization



Efficient state detection and cooling, even near B=0

Systematic Frequency Shifts

Effect	Correction (Hz at 1.06 PHz)	Uncertainty (Hz)
2 nd order Zeeman (B-field uncertainty)	I0, (typ., daily)	<0.1
2 nd order Zeeman (coefficient uncertainty)	0	0.01
$^2D_{rac{5}{2}}$ quadrupole shift	In progress	???
H-maser frequency		

Total fractional uncertainty: 10⁻¹⁴

•Negligible (at this level): ac stark shifts (laser, trap),

2nd order Doppler (micromotion, thermal),

background collision shifts (helium),

blackbody shift, fs comb noise.

•All effects except quadrupole should reduce below 10-18

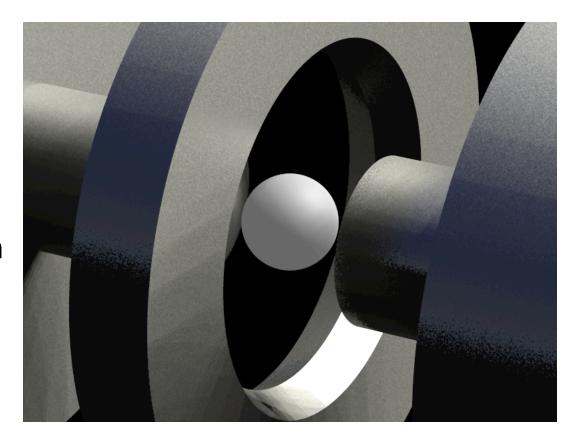


The Quadrupole Shift

•Shift due to coupling of ${}^2D_{5/2}$ -state electric quadrupole moment with stray E-field gradients

For ideal spherical rf trap:

- No static electric fields
- Transition frequency independent of quantization axis orientation



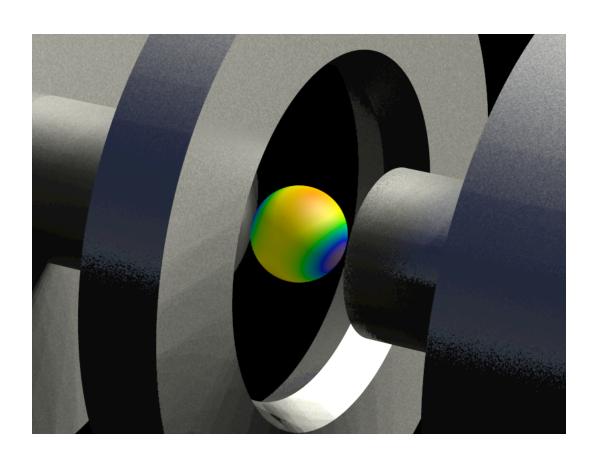


The Quadrupole Shift

•Shift due to coupling of ${}^2D_{5/2}$ -state electric quadrupole moment with stray E-field gradients

For real spherical rf trap:

- •Electric patch potentials
- •Transition frequency is a function of quantization axis orientation





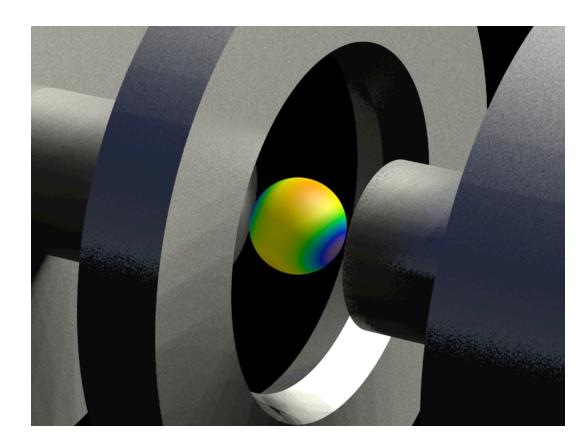
Evaluating Quadrupole Shift

•Quadrupole shift averages to zero over three orthogonal quantization axes [W. M. Itano, J. Res. NIST 105, 829 (2000)]

•Magnitude expected to be under 10^{-15} (1 Hz)

$$10^3 \text{ V/cm}^2 \Longrightarrow \approx 1 \text{ Hz}$$

•Need stable flywheel for this measurement!



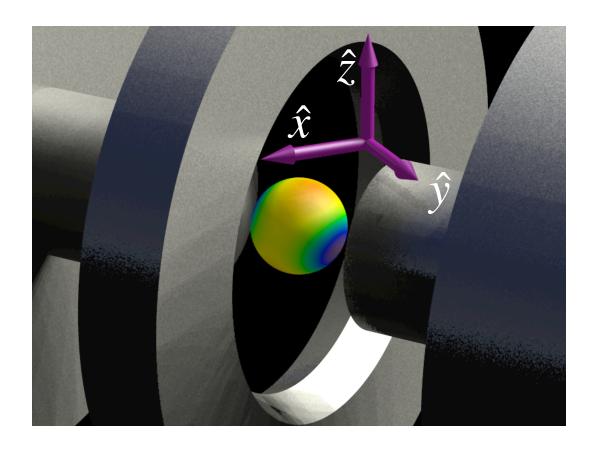
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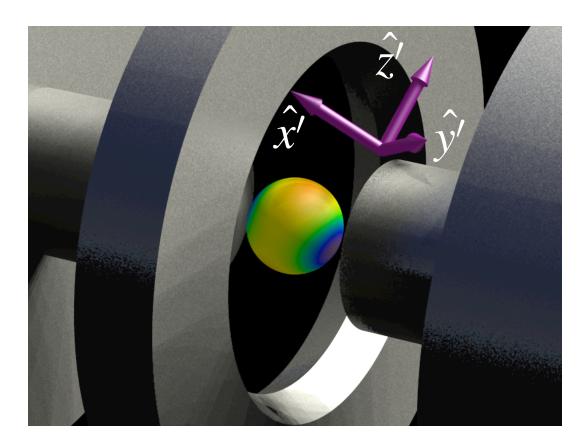
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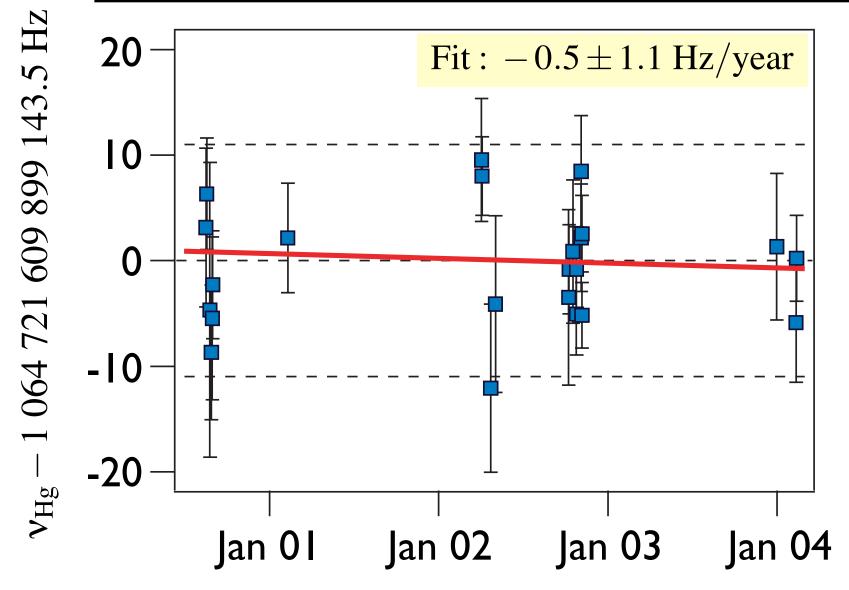
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Absolute frequency measurements



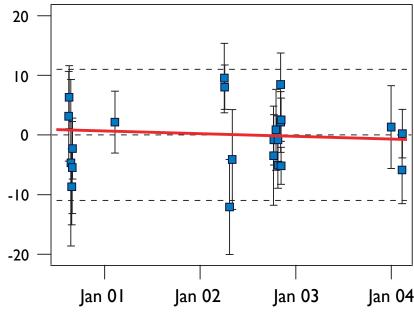
Absolute frequency measurements

- •No significant long-term change observed over three years.
 - -Fluctuations comparable to that of Cs fountain
 - -Promising for testing of stability of fundamental constants

No significant change with different traps or

trap surfaces (Au, Mo)

-Suggests low quadrupole shift



Transition Frequencies

•Clock comparisons may provide the best test of present-day changes in physical constants.

For example, look at Hg vs Cs, express transition frequencies as:

$$\nu_{\rm Hg} \approx R_{\infty} c F_{\rm Hg}(\alpha),$$

$$\nu_{\rm Cs} \approx g_{\rm Cs}(m_e/m_p) \alpha^2 R_{\infty} c F_{\rm Cs}(\alpha)$$

$$\frac{\alpha \frac{\partial}{\partial \alpha} (\ln F_{\rm Hg}(\alpha)) \approx -3.2}{\alpha \frac{\partial}{\partial \alpha} (\ln F_{\rm Cs}(\alpha)) \approx +0.8} \implies \frac{\mathbf{v}_{\rm Cs}}{\mathbf{v}_{\rm Hg}} \approx g_{\rm Cs}(m_e/m_p) \alpha^{6.0}$$

V.A. Dzuba, V.V. Flambaum, and J. K. Webb, PRA **59**, 230 (1999).

S. Bize et al., PRL **90**, 150802 (2003).

Test stability of this ratio

Results

- •With 10⁻¹⁴ accuracy and 3 years measurement, fractional change in ν_{Cs}/ν_{Hg} is $\leq \pm 6 \times 10^{-15} yr^{-1}$
- •If all variation in v_{Cs}/v_{Hg} is due to $\alpha,$ constrain present-day variation of α :

$$|\dot{\alpha}/\alpha| \le 1 \times 10^{-15} \mathrm{yr}^{-1}$$

•Look at measurements with other species to tighten constraints.

Other recent Results

 $\cdot v_{Rb}/v_{Cs}$ (microwave) Measurement interval: 4 years

$$\frac{\Delta(\nu_{Rb}/\nu_{Cs})}{(\nu_{Rb}/\nu_{Cs})} = (0.2 \pm 7.0) \times 10^{-16} \text{ yr}^{-1}$$

H. Marion et al., PRL **90**, 150801 (2003).

 $\cdot \nu_H / \nu_{Cs}$ (Optical) Measurement interval: 4 years

$$\frac{\Delta(\nu_{\rm H}/\nu_{\rm Cs})}{(\nu_{\rm H}/\nu_{\rm Cs})} = (3.2 \pm 6.4) \times 10^{-15} \text{ yr}^{-1}$$

M. Fischer et al., arXiv:physics/0312086

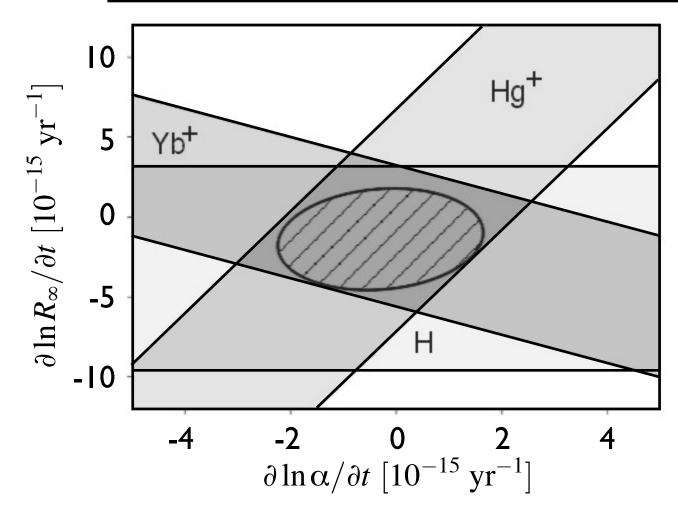
 $\cdot \nu_{Yb^+}/\nu_{Cs}$ (Optical) Measurement interval: 3 years

$$\frac{\Delta(\nu_{Yb^+}/\nu_{Cs})}{(\nu_{Yb^+}/\nu_{Cs})} = (-1.2 \pm 4.4) \times 10^{-15} \text{ yr}^{-1}$$

E. Peik et al., arXiv:physics/0402132



Limits with other standards



H M. Fischer et al., arXiv:physics/0312086

Yb+ E. Peik et al., arXiv:physics/0402132

$$\frac{\partial \ln \alpha}{\partial t} = (-0.3 \pm 2.0) \times 10^{-15} \text{ yr}^{-1} \qquad \frac{\partial \ln R_{\infty}}{\partial t} = (-1.5 \pm 3.2) \times 10^{-15} \text{ yr}^{-1}$$

Present work and Outlook

- Improvements to accuracy:
 - Reduced operating B-field, added shielding,
 B-field coils installed near ion trap
 - Proceed with evaluation of quadrupole shift;
- Improvements to stability:
 - Quench D-state (reduce dead time)
- Comparison of two Hg+ clocks
- Other optical-optical clock comparisons (Ca,Sr,Yb,Al+)

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